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In-Flight Magnetometer Calibration and Attitude Determination for Near-Earth Spacecraft

G.M. Lerner* and M.D. Shuster†

Computer Sciences Corporation, Silver Spring, Md.

A linear-regression algorithm is used to simultaneously determine magnetometer biases, misalignments, and scale factor corrections, as well as the effect on the measured magnetic field of magnetic control systems. This algorithm has been applied to data from the Seasat-1 and the Applications Explorer Mission-1/Heat Capacity Mapping Mission (AEM-1/HCMM) spacecraft. Results show that complete in-flight calibration as described here can significantly improve the accuracy of attitude solutions obtained from magnetometer measurements. For the HCMM orbit, the attitude accuracy is shown to be limited by the field-model error, which is shown to be 0.25-0.5 deg rms.

I. Introduction

AGNETOMETERS are widely used for attitude determination and control. As part of an attitude control system, a three-axis magnetometer measures the strength and direction of the Earth's magnetic field and can be used to compute electromagnetic torquing system commands to control the spacecraft angular momentum. Magnetic torquing can be used directly to control nutation, to precess the spacecraft angular momentum vector, or to maintain the speed of momentum wheels within prescribed limits

For attitude control, a crude magnetometer will suffice. A momentum management control law, which was implemented in the Application Explorer Mission, uses only the sign of the magnetic field vector components. In contrast, for attitude determination, the best magnetometers are inadequate when attitude accuracies better than 0.5 deg/axis are required.

The accuracy of attitude solutions from magnetometer measurements cannot exceed that of the magnetic field model. Errors in geomagnetic field models [e.g., IGRF75 (Ref. 2)] frequently exceed 0.5 deg for a 600-800-km orbit. ³ Even this attitude determination accuracy cannot be achieved without correction for numerous sources of additional error. These sources include bias magnetic fields produced in the spacecraft, internal misalignment or miscalibration of the magnetometer system, external misalignment of the magnetometer system relative to the spacecraft reference axes, and calibration errors in the conversion of analog magnetometer measurements to digital telemetry data. Zeromean error sources, such as noise on the analog signal and the finite size of the least significant telemetered bit, also need to be considered.

Experience with magnetometers flown onboard Small Astronomy Satellite-1 (SAS-1), Atmosphere Explorer-3 (AE-3), SAS-3, and the two spacecraft treated here, Seasat-1 and AEM-1/HCMM, has shown that the postlaunch determination of magnetometer biases is necessary for

spacecraft that use magnetometers for attitude control. On HCMM, an uncalibrated magnetometer was used to provide coarse (about 5 deg) attitude solutions during attitude acquisition, but was of limited use in support of daily experimental operations.

This paper describes an algorithm for the in-flight calibration of three-axis magnetometer systems. These methods have been applied to flight data for Seasat-1 and AEM-1/HCMM. Results show that in-flight calibration similar to the type presented here can significantly improve attitude accuracy. For AEM-1/HCMM, the improvement is dramatic.

The magnetometer calibration algorithm, which is presented here, requires that a reference three-axis attitude be available. This is generally the case. Near-Earth spacecraft with moderate attitude accuracy requirements (approximately 1 deg) are usually equipped with two-axis sun sensors and Earth horizon scanners as prime attitude sensors. Unfortunately, this sensor combination cannot provide complete attitude information during orbit night. By suitably calibrating the magnetometer, however, using data collected during orbit day, the loss of attitude information during orbit night can be greatly reduced.

Estimates of attitude accuracy using magnetometers based on in-flight experience are not currently available in the literature. For the AEM-1/HCMM spacecraft, the sources and magnitude of attitude errors are examined in detail. An estimate of the field model accuracy on the basis of attitude estimates is also given.

II. Bias Determination Equations

The availability of a source of attitude knowledge independent of the magnetometer data is assumed. This source may be from any combination of star sensors, Sun and Earth sensors, or a dynamic model. The model chosen for bias determination is

$$A(t)B_0(t) = (I+S)B(t) + b + TD(t)$$
(1)

A(t) is the 3×3 attitude matrix at time t that transforms vectors from reference to body coordinates. Since both spacecraft to be considered are Earth-pointing, it will be convenient to choose orbital coordinates (\hat{Z} =nadir, \hat{Y} =negative orbit normal) as the reference system. $B_0(t)$ is the Earth's magnetic field in reference (orbital) coordinates and B(t) is the Earth's magnetic field in the spacecraft body coordinates based on nominal (preflight) calibration of the magnetometer. D(t) is a telemetered control vector (e.g., the spacecraft control magnetic dipole), which is assumed to linearly affect the magnetometer data; b is a bias vector, to be

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^{*}Senior Computer Scientist, Attitude Systems Operation, System Sciences Division.

[†]Senior Member of the Technical Staff, Attitude Systems Operation, System Sciences Division; currently, Staff Scientist, Business and Technological Systems, Inc., Seabrook, Md. Member AIAA.

determined; S is a 3×3 scale-factor/misalignment matrix, to be determined; T is a 3×3 matrix relating the control vector to the magnetometer data, to be determined; and I is the 3×3 identity matrix.

The matrix I+S is the alignment/scale-factor matrix. If I+S is an orthogonal matrix, the three magnetometer axes are orthonormal and coherently misaligned to the spacecraft reference axes. Nonzero diagonal elements of S are indicative of errors in the magnetometer axis scale factor; that is, the constant relating magnetometer output to magnetic field. In general, the nine elements of S are small and unrelated and include both alignment and calibration errors.

The desired solutions for S, T, and b are those that minimize the loss function

$$L = \sum_{i=1}^{N} a_i |\boldsymbol{H}_i - \boldsymbol{b} - S\boldsymbol{B}_i - T\boldsymbol{D}_i|^2$$
 (2)

where

$$\boldsymbol{H}_i = A(t_i)\boldsymbol{B}_0(t_i) - \boldsymbol{B}(t_i) \tag{3a}$$

$$\boldsymbol{B}_i = \boldsymbol{B}(t_i) \qquad \boldsymbol{D}_i = \boldsymbol{D}(t_i) \tag{3b}$$

and the a_i are nonnegative weights associated with the measurements at time t_i and normalized so that

$$\sum_{i=1}^{N} a_i = 1 \tag{4}$$

Straightforward minimization of this loss function leads to the equations

$$S\langle B|B\rangle + T\langle D|B\rangle = \langle H|B\rangle \tag{5a}$$

$$S\langle B|D\rangle + T\langle D|D\rangle = \langle H|D\rangle \tag{5b}$$

$$b = \langle H \rangle - S \langle B \rangle - T \langle D \rangle \tag{5c}$$

where the sampled mean of a vector has been written as

$$\langle F \rangle = \sum_{i=1}^{N} a_i F_i \tag{6}$$

and the sampled covariance matrix of two vectors as

$$\langle F | G \rangle = \langle F G^T \rangle - \langle F \rangle \langle G^T \rangle \tag{7}$$

Equations (5a) and (5b) may be reduced further to obtain

$$S = [\langle H | B \rangle - \langle H | D \rangle \langle D | D \rangle^{-1} \langle D | B \rangle]$$

$$\times [\langle B | B \rangle - \langle B | D \rangle \langle D | D \rangle^{-1} \langle D | B \rangle]^{-1}$$
(8)

and an identical expression for T with B and D interchanged. When the term linear in the control vector is not included in the loss function, the simpler result is obtained

$$S = \langle H | B \rangle \langle B | B \rangle^{-1} \qquad b = \langle H \rangle - S \langle B \rangle \tag{9}$$

III. Application to Seasat-1

Calibration Results

Four orbits of Seasat-1 data (325 samples at 1-min. intervals) were analyzed. Reference attitudes were computed using data from an Adcole sun sensor and an ITHACO infrared horizon scanner, which provided an attitude accuracy of 0.3 deg/axis (3 σ).^{4,5} The results for the magnetometer

misalignment/scale-factor matrix and the bias vector are

$$b = \begin{bmatrix} 71.5 \\ -28.1 \\ -61.2 \end{bmatrix} \text{ mG}$$
 (10)

$$S = \begin{bmatrix} 0.021 & 0.016 & 0.004 \\ 0.013 & -0.022 & -0.004 \\ 0.007 & -0.003 & 0.002 \end{bmatrix}$$
 (11)

The rms residual error in the magnetic field using these calibration coefficients is 3.7 mG.

The exclusion of a term linear in the control vector is justified in the case of Seasat-1, since the control electromagnets are separated from the magnetometers by 6 m. Thus, a control magnetic dipole of 10⁵ pole-cm can create a magnetic field of no greater than 1.7 mG at the magnetometers.

The data indicate that the bias vector b is well defined and independent of the size of the data base, and that the magnitude of b, 98 mG, is about 40% of the Earth's field at the equator (for Seasat at 800 km). The reason for this large bias is unclear; however, it is clear that attitudes computed from Seasat magnetometers data are useless without in-flight calibration. The matrix S is not observable using data from a single orbit, although four orbits of data provide reasonably consistent results for all the components except S_{32} . The diagonal components of S indicate scale-factor calibration errors for the X and Y axes of up to 2%. The off-diagonal elements of S are equivalent to alignment errors of up to 1 deg. The elements of S are consistent with values expected from the specified alignment and calibration accuracy. Because S is not skew-symmetric, the misalignment of the vector magnetometer relative to the body reference axes is not a simple rotation. Rather, S contains information on internal alignment, external alignment, and cross talk. The conference report⁶ from which this report is drawn contains results for these coefficients using several different sample sets.

The quality of the fit of 12 parameters to as many as 975 data items may be judged by the rms deviation between the modeled and measured field components. Table 1 lists the various sources of error that contribute to the observed rms error of 3.7 mG. The known error is 2.6 mG, implying an unknown error of $\sqrt{(3.7)^2 - (2.6)^2} = 2.6$ mG. This unknown error could include such sources as noise on the analog magnetometer output, undetected bit errors, and currents associated with the power subsystem (the solar arrays and power cables are located near the magnetometer). Note that for GEOS-3 in a similar orbit, the rms error was 1.5 mG, of

Table 1 Sources of unmodeled error for Seasat

Source	Magnitude	Contribution to rms error, mG	Reference
Magnetometer LSB	4.4 mG	1.3	4
Reference attitude error	0.1 deg	0.5	5
Electromagnet activity	10 ⁵ pole-cm (3 axes)	1.7	this work
Field model error	IGRF 75	1.4	2,3
Anticipated rms error	•••	2.6	

which all but 1.0 mG resulted from the least significant bit (LSB) size.⁷

Seasat Yaw Determination Accuracy

The magnetometers may be used to measure yaw angle during orbit night and the portion of the orbit during which the sun is not visible to any of the four sun sensors. During the last half of August 1978, and through September 1978, sun sensor data were available only in the Southern Hemisphere for about 20% of the orbit (30-35% of the orbit was in darkness), and magnetometer data were needed to supplement interpolation methods for yaw determination. Because of the Seasat spacecraft's failure on October 10, 1978, after a 106-day mission, it is particularly important to provide the best possible yaw angle data during that short period.

To the first order in the attitude angles, yaw may be expressed as a function of the magnetometer data and known roll and pitch angles as

$$y = \frac{B_{02}(B_1 + pB_3) - B_{01}(B_2 - rB_3)}{B_{01}^2 + B_{02}^2}$$
 (12)

where B is the measured magnetic field in the body after inflight calibration and B_0 is the magnetic field in orbital (reference) coordinates. The quantities y, r, and p (yaw, roll, and pitch) are a 3-1-2 set of Euler rotation angles from orbital to body coordinates.

To obtain Eq. (12), it may be noted that the magnetic field vector in the spacecraft body frame is related to the field vector in orbital components by

$$\mathbf{B} = \begin{bmatrix} 1 & y & -p \\ -y & 1 & r \\ p & -r & 1 \end{bmatrix} \mathbf{B}_0 \tag{13}$$

Yaw is overdetermined by Eq. (13), and the pseudoinverse solution leads to Eq. (12).

The expected yaw variance, σ_y^2 , may be written as a function of the variance of the magnetometer measurements, σ_B^2 , and the variance in the Euler angles, σ_θ^2 , assumed to be the same for pitch and roll. Differentiating Eq. (12), squaring, and taking the expectation value leads to

$$\sigma_y^2 = \frac{\sigma_B^2 + B_{03}^2 \sigma_\theta^2}{B_{01}^2 + B_{02}^2} \tag{14}$$

For the circular Seasat orbit at an altitude of 790 km with an inclination of 108 deg, the yaw variance is a minimum at the equator, where the standard deviation is 0.92 deg. The maximum yaw error occurs near the magnetic poles, where the denominator of Eq. (14) vanishes. There, the largest possible error for the Seasat orbit is 11 deg.

In addition, the error associated with the size of the least significant bit (LSB) in the magnetometer telemetry (which is equivalent to 4.4 mG) must be considered. This leads to yaw angle quantization errors from 0.4 deg (from X axis quantization) to 1.0 deg (from Y axis quantization) at the equator, and correspondingly larger errors at the poles. In principle, LSB errors have zero mean and can be removed by the filtering or smoothing of yaw solutions. In practice, however, the LSB size is an important source of systematic yaw error because the magnetometer measurements change slowly at the orbit rate.

Figure 1 shows a comparison of yaw angle solutions computed using horizon scanner data and either sun sensor or magnetometer data. The sun sensor yaw solutions are believed to be accurate to ± 0.3 deg; the magnetometer yaw solutions

were computed using mean biases over the orbit, T=S=0, $b=(69.3, -30.3, -61.9)^T$ mG, and averaged over 120-s intervals to reduce the effects of magnetometer data digitization and noise. The figure indicates that, when mean biases are used, the magnetometer yaw solutions are worse than the trivial solution yaw=0 over the entire orbit, even near the ascending and descending nodes where the errors should be the smallest.

Figure 2 shows a comparison of the sun sensor and magnetometer yaw solutions using the in-flight calibration parameters given by Eqs. (10) and (11). In the figure, the dashed line indicates the sun sensor solution, the broken lines denote individual magnetometer solutions, and the dotted line is the magnetometer yaw solution obtained by averaging over 120-s intervals. The discontinuities in the yaw solutions from the raw magnetometer data reflect bit changes in the telemetry of the magnetometer readings. The figure shows that the inflight calibration substantially improves the accuracy of the magnetometer solutions, although the solution yaw = 0 is still the better estimate over much of the orbit. The yaw angle accuracy and the yaw angle LSB in Fig. 2 are in good agreement with the estimates presented earlier.

IV. Application to AEM-1/HCMM

Calibration Results

The HCMM mission is described in Ref. 9. The attitude determination hardware consists of an infrared Earth horizon sensor, 3 two-axis sun sensors, and a three-axis fluxgate magnetometer. The magnetometer is part of the magnetic attitude and momentum control system and is located within a meter of three 10,000 pole-cm electromagnetic torquing coils. The proximity of the coils and magnetometers can result in a bias of approximately 20 mG, which is removed, in part, by electronic compensation. This compensation may be thought of as subtracting bias voltages, proportional to the dipole currents, from the magnetometer voltages. The compensation is desired to reduce the effective dipole bias to a value less than 1 mG. Thus, as in the case of Seasat-1, we assume that $D_i = 0$.

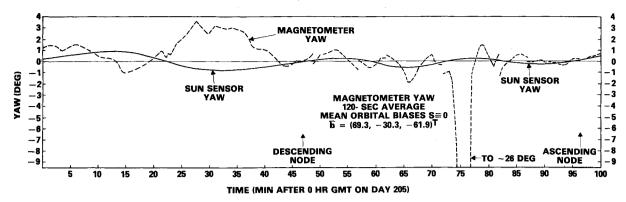
Magnetometer and sun sensor data were used to provide coarse attitude information during the early orbits of the mission. Subsequent to the initiation of normal experimental operations, however, magnetometer data were found to be of little use in meeting the experimenter's attitude accuracy requirements of 0.5, 0.7, and 2.0 deg for pitch, roll, and yaw, 10 respectively, because the HCMM software was capable of solving only for b in Eq. (2), under the assumption that S = T = 0. The value obtained for b_1 varied from data pass to data pass by as much as ± 10 mG.

The results of the complete in-flight calibration are

$$b = \begin{bmatrix} -11.1 \\ 16.1 \\ -1.5 \end{bmatrix} \text{ mG}$$
 (15)

$$S = \begin{bmatrix} 0.066 & 0.005 & -0.003 \\ -0.019 & 0.014 & 0.018 \\ 0.024 & -0.004 & -0.007 \end{bmatrix}$$
 (16)

It is noted that the rms difference between the model field and the calibrated magnetometer reading is 2.2 mG, considerably smaller than the value of 3.7 mG obtained for Seasat-1. The smaller error was obtained despite a poorer attitude reference and greater electromagnetic effects. Table 2 lists the error sources for HCMM and shows that the observed rms residual may be accounted for fully by the known error sources. This



Comparison of sun sensor and magnetometer yaw angle solutions for day 205 using mean bias for the orbit.

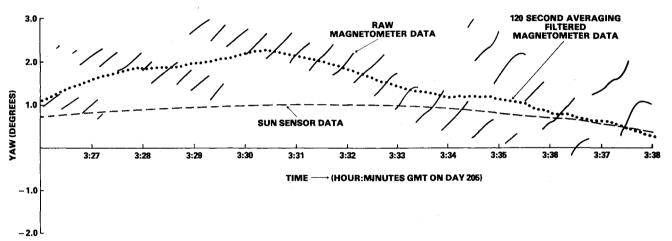


Fig. 2 Comparison of yaw solutions using sun sensor and magnetometer data with complete magnetometer calibration.

result indicates that the electronic compensation works well for HCMM.

The 7% error in the X-axis scale factor is the main reason that the apparent X-axis bias varied from pass to pass and that the uncalibrated magnetometers could not provide useful attitude data. Misalignments of 1-1.4 deg and X- and Y-axis biases of -11 and +16 mG also contributed to the magnetometer data problems.

The accuracy of the calibrated magnetometer data may be judged by examining scalar (i.e., attitude-independent) parameters. The field model residual

$$\epsilon_{M} = \left\{ \frac{I}{N} \sum_{i=1}^{N} \left[M_{M}(t_{i}) - M_{C}(t_{i}) \right]^{2} \right\}^{1/2}$$
 (17)

is the rms difference between the measured field magnitude. $M_M(t_i)$, and the model field magnitude, $M_C(t_i)$, at time t_i . If the only source of error were the 4.7 mG LSB of the telemetered magnetometer data, the rms residual would be

$$\sigma_{\epsilon_M} = 4.71/\sqrt{12} = 1.36 \text{ mG}$$
 (18)

For HCMM, ϵ_M ranges from 1.65 to 5.46 mG; however, in most cases, ϵ_M is less than 3 mG. Detailed flight data are given in Table 5 of Ref. 6. For the majority of data (about 90% of all cases), the unmodeled rms magnetic field residual is

$$\delta \epsilon_M = [(2.2)^2 - \sigma_{\epsilon_M}^2]^{1/2} = 1.7 \text{ mG}$$
 (19)

which may be fully accounted for by errors in the field model and electromagnetic activity (see Table 2).

The reason for the large unmodelable residuals in 10% of the cases is unknown. These cases all occurred at night over the region of primary experimental interest, indicating that bias fields produced by onboard hardware may be responsible.

Angular errors in the direction of the magnetic field may be estimated by

$$\delta\theta_M \approx \delta\epsilon_M/M \tag{20}$$

where M varies from 230 mG near the equator to 460 mG near

Table 2 Sources of unmodeled error for HCMM

Source	Magnitude	Contribution to rms error, mG	Reference
Magnetometer LSB	4.7 mG	1.4	9
Reference attitude error	a	1.1	11
Electromagnet activity	ь	<1	9
Field model error	IGRF 75	1.0-1.5	2,3
Magnetometer noise	Negligible	•••	•••
Anticipated total rms error		2.0-2.5	

 $[^]a$ During sunlit passes, the rms pitch and roll error estimates are 0.15 deg and 0.12 deg, respectively. 11 The rms yaw error during sunlit passes is estimated from Figs. 4.54-4.57 of Ref. 9 as 0.3 deg. b Electronic compensation for large ($\sim\!20$ mG) magnetometer biases is used

on the AEM spacecraft. Residual effects are believed to be less than 1 mG.

the poles. This implies an rms angular error in the range

$$0.42 \deg \ge \delta \theta_M \ge 0.21 \deg \tag{21}$$

with the larger errors near the equator. Attitude errors, however, may be smaller near the equator because the nadir and the field vectors are normal near the equator and parallel near the poles.

The angular errors may also be estimated from rms residuals of the form

$$\Delta\theta_{XY} = \left\{ \frac{1}{N} \sum_{i=1}^{N} \left[\cos^{-1} \left(\hat{X}_{M}(t_{i}) \cdot \hat{Y}_{M}(t_{i}) \right) - \cos^{-1} \left(\hat{X}_{C}(t_{i}) \cdot \hat{Y}_{C}(t_{i}) \right) \right]^{2} \right\}^{1/2}$$
(22)

where \hat{X} and \hat{Y} denote the direction of any two of the field M, sun S, and nadir E reference vectors. The subscripts M and C denote the measured and computed values for these angles. As for ϵ_M , $\Delta\theta_{XY}$ is composed of (zero-mean) noise and LSB errors in the measured field, sun, and nadir vector directions and systematic sensor errors. Errors in the sun and spacecraft ephemerides are negligibly small so that errors in \hat{M}_C contribute to errors in $\Delta\theta_{SM}$ and $\Delta\theta_{EM}$, but not in $\Delta\theta_{SE}$, which may be used to estimate errors in the measured sun, \hat{S}_M , and nadir, \hat{E}_M , vectors.

HCMM flies 3 two-axis sun sensor heads (sensors 1, 2, and 3) each with a 128×128 deg field of view and a 0.5-deg LSB (Ref. 9). Sensors 2 and 3 were used to calibrate the horizon scanner data with an algorithm¹¹ that, in essence, assumes that the sun sensor orientation in the spacecraft is known and calibrates the infrared horizon scanner by minimizing $\Delta\theta_{SE}$. Thus, for the purpose of attitude determination, sun sensors 2 and 3 and the horizon scanner define the HCMM attitude reference axes. Data from these three sensors are assumed to be free of systematic errors.

The zero-mean rms noise on the measured sun and nadir vectors, assumed equal, is estimated to be $\Delta\theta_{SE}/\sqrt{2}\approx 0.17$ deg. The rms error in the measured field direction, composed of zero-mean and systematic errors, may be estimated by either $\Delta\theta_{SM}$ or $\Delta\theta_{EM}$, reduced by the 0.17-deg rms error in the measured sun or nadir vector. These estimates are 0.31 and 0.47 deg, respectively, and are consistent with Eq. (20) and the previous estimates of the field model accuracy cited in the Introduction. In conclusion, the in-flight calibration of the HCMM magnetometers has removed most of the systematic measurement error, leaving only a systematic rms field model error of 0.25-0.5 deg and a zero-mean rms measurement error of 0.3 to 0.6 deg.

HCMM Yaw Determination Accuracy

The relationship among the residual rms errors, $\Delta\theta_{SM}$ and $\Delta\theta_{EM}$, the rms error in the direction of the magnetic field, and errors in yaw derived from the calibrated magnetometer data are not easily shown analytically. These relations depend on the sensor accuracies and the reference vector geometry.

If we assume that the rms errors in the measurement of \hat{M} parallel and normal to \hat{E} are equal, the rms error is given by $\Delta\theta_{EM}$ reduced by the error in the measurement of \hat{E} . If the 0.1-0.2-deg error in \hat{E} is neglected, then $\Delta\theta_{EM}$ may be used as a conservative estimate of the rms yaw error. This means that $\Delta\theta_{EM}$ ranges from 0.32 to 0.96 deg, with an overall rms value of 0.53 deg.

Equation (14) with $\sigma_B = 1.7$ mG, $\sigma_\theta = 1.7$ deg, and

$$\boldsymbol{B}_{0} = 230 \begin{bmatrix} \cos \lambda \\ 0 \\ 2\sin \lambda \end{bmatrix} \quad \text{mG} \tag{23}$$

may also be used to estimate the systematic rms yaw angle errors. λ is the subsatellite latitude when the spacecraft is traveling north. This leads to typical results of from 0.5 to 1.24 deg, with the highest values occurring near the magnetic poles, where the nearly colinear reference vectors make attitude accuracy poorer. Except for regions very near the poles, values from 0.50 to 0.80 deg are the rule.

V. Conclusions

An efficient and easily implemented algorithm for the inflight calibration of magnetometers has been presented. The algorithm has been applied to Seasat-1 and AEM-1/HCMM flight data. For Seasat-1, large errors in the preflight calibration were determined, but a 3.7 mG residual, unmodeled bias on the data severely limited the obtainable yaw attitude accuracy. In-flight calibration of the HCMM magnetometers, however, removed large preflight calibration errors, including a 7% X-axis scale-factor error, and enabled the use of magnetometer yaw data in support of HCMM experimental objectives. Errors in the geomagnetic field model were estimated and found to be 1-1.5 mG (rms) in magnitude and 0.5 deg (maximum) in direction.

Acknowledgments

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